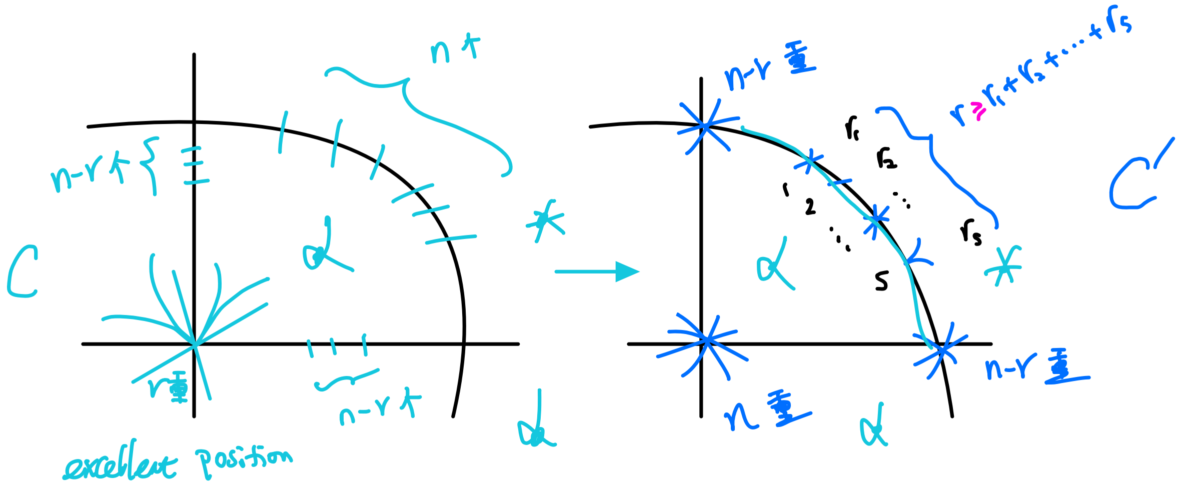


Fact: Denote $g^*(C) := \frac{(n-1)(n-2)}{2} - \sum_{P\in C} \frac{r_P(r_P-1)}{2}$. Then

$$\boxed{g^*(C') := g^*(C) - \sum_{i=1}^{s} \frac{r_i(r_i-1)}{2}}$$

$r_i \neq 1$

$\Downarrow$

$$\boxed{g^*(C') < g^*(C)}$$

Pf: $\deg(C') = 2n-r$

Singular pts on C' :
- $[0:0:1]$ & $[0:1:0]$ & $[1:0:0]$
- $Z=0$ & $X\neq 0, Y=0$.
- the one coming from $C\setminus[0:0:1]$

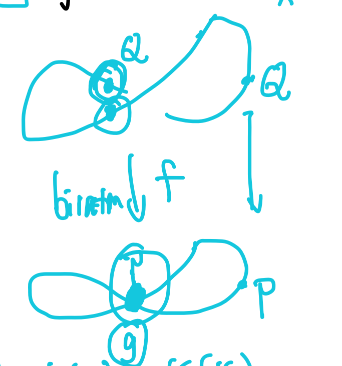
$$g^*(C') = \frac{(2n-r-1)(2n-r-2)}{2} - \frac{n(n-1)}{2} - 2\cdot\frac{(n-r)(n-r-1)}{2} - \sum_{P\in C\setminus[0:0:1]} \frac{r_p(r_p-1)}{2}$$

$(g, \#\{\text{non ordinary pts}\})\downarrow$

Thm 3 C = proj curve. Then $\exists !$ nonsingular proj. curve X & birational morphism $f : X \twoheadrightarrow C$.
← nonsingular model of C

DVR
$\downarrow$
$\mathcal{O}_Q(X)$

X



Notation: $f : X \to C$ ← plane curve. $f(Q) = P \in C$

$\mathrm{ord}_Q(G) := \mathrm{ord}_Q(g)$

$C \hookrightarrow \mathbb{P}^2 \hookleftarrow G$

$g \in K(C) = K(X)$

$G_* \bmod I(C) = g \in K(C)$

Def: 1) pts of X will be called places of C (or of K)
2). A place Q is centered at P if $f(Q) = P$

$\mathrm{Spec}\, \mathcal{O}_K$
K

$= \mathcal{O}_P(\mathbb{P}^2)/(F_*, G_*) \cong \left(\mathcal{O}_P(\mathbb{P}^2)/F_*\right) / \left((F_*, G_*)/F_*\right)$

$\mathcal{O}_P(F)/(g)$

$P \in F$ simple $\Rightarrow I(P, F \cap G) = \mathrm{ord}_P^F(G)$

what about when $P \in F$ is not simple?

Prop 2. C = Irr. proj. plane curve, $P \in C$, $f : X \to C$ nonsingular model
G = plane curve (possibly reducible). Then

$$I(P, C \cap G) = \sum_{Q \in f^{-1}(P)} \mathrm{ord}_Q(G)$$

$\{Q_1, \ldots, Q_s\} \subset X \supset f^{-1}(P)$
$\downarrow \qquad \downarrow f$
$P \qquad C \ni \{P\}$